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Technical note: stable and unstable reactors

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Abstract. It is well known that a reactor is stable if the core reactivity decreases with the core power. This is the case for many types of reactors, including the PWR. However, this was not the case for the RBMK (Reaktor Bolshoy Moshchnosti Kanalniy) which could be unstable at low power. What does it mean precisely? By using a 2×2 system of non-linear ordinary differential equations we show that naturally (i.e. without using the control rods), with the same reactivity injection, if the initial power is lowered, then the final power may be higher, which is a rather unusual behaviour.

1 Introduction

On the web, we find https://www.nuclear-power.com/nuclear-power/reactor-physics/reactor-dynamics/reactor-stability/ which states:

"*The response of a reactor to a change in temperature (i.e., the overall reactor stability) depends especially on the algebraic sign of* α_T. *A reactor with negative* α_T *is inherently stable to changes in its temperature and thermal power, while a reactor with positive* α_T *is inherently unstable*."

In the present paper, we consider the case of a reactor for which α_T is positive at low power and negative at large power. This case is of interest since this was how the RBMK behaved at the time of the Chernobyl accident.

Let k be the neutron effective multiplication factor.

Let $\rho = \frac{k-1}{k}$ denote the reactivity.

Obviously when $\rho > 0$ (resp. $\rho < 0$) the reactor is supercritical (resp. subcritical). In other words, the linear theory predicts that the reactor is unstable. It also tells us that when the reactor is critical (i.e. $\rho = 0$) to increase its power, one has to make a positive reactivity injection $\rho_0 > 0$, e.g. by moving up the control rods, and then, when the power has reached the desired power level, a *negative* reactivity injection $-\rho_0$ e.g. by moving down the control rods where they were before.

This is not, as we shall see, how it works in a real nuclear reactor where there is a *reactivity feedback*.

Let n denote the normalized reactor power such that $n = 1$ at nominal power, the linear theory predicts that

$$\frac{\mathrm{d}}{\mathrm{d}t} n = \frac{1}{\tau} \rho_0 n \tag{1}$$

where τ is the effective neutron lifetime, which means that $n(t)$ increases or decreases exponentially.

In a PWR, when the power increases, the fuel temperature increases inducing the Doppler effect. The moderator temperature also increases, inducing a moderator effect.

In other words, there is a negative reactivity feedback $\alpha(n) < 0$ such that

$$\frac{\mathrm{d}}{\mathrm{d}n} \rho = \alpha(n). \tag{2}$$

Therefore ρ is not a constant and we should replace (1) by

$$\frac{\mathrm{d}}{\mathrm{d}t} n = \frac{1}{\tau} \rho n. \tag{3}$$

Note that a point reactor model is used for n; τ takes into account the delayed neutrons; α takes into account both moderator and Doppler effects and is supposed to depend only on n.

In view of

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} \bigg/ \frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}n} \rho = \alpha(n)$$

we obtain

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \quad \frac{1}{\tau} \alpha(n) \rho n. \tag{4}$$

Finally, we have to study the 2×2 nonlinear differential system:

$$\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \rho \\ n \end{pmatrix} = \begin{pmatrix} \alpha(n) \rho n / \tau \\ \rho n / \tau \end{pmatrix} \tag{5}$$

$$n(0) = n_0; \quad \rho(0) = \rho_0. \tag{6}$$

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Note that we can exclude the particular case $n_0 = 0$ which is not of interest.

2 Reactor stability

If, for a while, we consider ρ as a function of n, we have the differential equation (2) which should be complemented by the "initial" condition

$$\rho(n_0) = \rho_0. \tag{7}$$

So that we have

$$\rho = \rho(n_0, \rho_0, n) = \rho_0 + \int_{n_0}^{n} \alpha(m)\mathrm{d}m \tag{8}$$

which holds both for $n > n_0$ and for $n < n_0$.

The reactivity injection ρ_0 can be positive (in which case the reactor power increases) or negative (in which case it decreases).

The reactor stability requires, as we shall see, that there exists α_* such that:

$$\alpha(n) \leq \alpha_* < 0 \, \forall \, n > 0. \tag{9}$$

Indeed, with such an assumption, for all starting point $\{n_0, \rho_0\}$, the curve:

$$n \to \rho = \rho(n_0, \rho_0, n)$$

will cut the horizontal axis once and only once in the plane $\{n, \rho\}$. In other words, the equation

$$\rho(n_0, \rho_0, n_\infty) = 0 \tag{10}$$

has a unique solution. Note, however, that we may have $n_\infty < 0$.

The particular case where $\alpha(n)$ is constant has been considered in Reuss-Mercier [1].

In such a case, we have

$$\rho(n_0, \rho_0, n) = \rho_0 + \alpha(n - n_0)$$

so that the curve $n \to \rho = \rho(n_0, \rho_0, n)$ is a straight line.

In this section, we consider the general case described by (9).

Proposition 1. Under the above assumption (9), then $n(t) \to \max(0, n_\infty)$ when $t \to \infty$.

Proof. We divide (4) by ρ and (3) by n so that we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Log}\,|\rho| = \alpha(n)\frac{n}{\tau} \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Log}\,n = \frac{\rho}{\tau}. \tag{12}$$

1°/Case where $\rho_0 > 0$.

Equation (12) shows that $n(t)$ increases, at least in the neighborhood of $t = 0$, but can reach a maximum only if ρ vanishes which means $n(t) = n_\infty$.

As long as $n(t) < n_\infty$, we have $\rho(t) = \rho(n_0, \rho_0, n(t)) > 0$.

Since we have

$$\alpha(n(t))\, n(t) < \alpha_* n(t) < \alpha_* n_0$$

we deduce that $\mathrm{Log}\rho(t) \to -\infty$ if $t \to \infty$ so that $\rho(t) \to 0$.

The curve $n \to \rho = \rho(n_0, \rho_0, n)$ being continuous, we deduce that $n(t) \to n_\infty$.

2°/Case where $\rho_0 < 0$.

In an analogous way, (12) shows that $n(t)$ must be decreasing, at least in the neighborhood of $t = 0$ but can reach a minimum only if ρ vanishes, then if $n = n_\infty$.

Let us assume first that $n_\infty > 0$.

We have $n_\infty \leq n(t) \leq n_o$ and then $\alpha(n(t)).n(t) < \alpha_*.n(t) < \alpha_*.n_\infty < 0$.

Therefore $\mathrm{Log}\,|\rho(t)| \to -\infty$ and then $|\rho(t)| \to 0$ so that $n(t) \to n_\infty$.

Let us now assume that $n_\infty < 0$.

The curve $n \to \rho(n_0, \rho_0, n)$ cuts the vertical axis $n = 0$ in $\rho = \rho_* < 0$.

We deduce that $\rho(t)$ increases and that $\rho(t) \to \rho_*$

Since $\rho_0 \leq \rho \leq \rho_* < 0$ we deduce from (12) that $\mathrm{Log}\,n(t) \to -\infty$ and then $n(t) \to 0$.

With this result, we have shown that for a stable reactor, due to the reactivity feedback, to increase the power of the reactor, it is sufficient to make an appropriate reactivity injection ρ_0, but it is not necessary to make a negative reactivity injection equal to $-\rho_0$ after this positive reactivity injection.

3 Evaluation of $\alpha(n)$ for the RBMK

It is well known that, before the Chernobyl accident, there was a significant void effect on the RBMK. This void effect originated from the fact that graphite is used as a moderator and water as a coolant. Since light water absorbs thermal neutrons, if it disappears, the core reactivity increases.

More precisely, in [2, p.55] we find that the Doppler effect brings a negative reactivity of −1000 pcm and that the void coefficient may be as high as +2500 pcm. (This was the case just before the accident).

We assume that the Doppler effect is proportional to n (e.g. −100 pcm for $n = 0.1$).

To evaluate the coolant effect, we assume that

- the coolant pressure is 7 MPa, then the saturation temperature is 285.8 °C.
- As for the enthalpy, we have $h_{l,\mathrm{sat}} = 1267$ kJ/kg for saturated liquid $h_{v,\mathrm{sat}} = 2772$ kJ/kg for saturated steam.
- With obvious notations, we have $\rho_{l,\mathrm{sat}} = 739\,\mathrm{kg/m^3}$ and $\rho_{\mathrm{v,sat}} = 36\,\mathrm{kg/m^3}$.
- The associated specific volumes are $\tau_{l,\mathrm{sat}} = 1.35$ L/kg and $\tau_{v,\mathrm{sat}} = 27.8$ L/kg.
- The mass flow rate is $q = 10417$ kg/s.

The inlet enthalpy is adjusted in such a way that the saturation level is at $z_{\mathrm{sat}} = 1.855$ m (note that $H = 7$ m is the height of the core).

To obtain the values given in Table 1, we proceed in the following way.

Table 1. Contributions of the coolant and Doppler effects to reactivity.

n	Power (MW)	x out	Coolant (pcm)	Doppler (pcm)	Reactivity (pcm)	alfa pcm/MW
0.05	160	0.007	117	−50	67.4	0.42
0.1	320	0.015	208	−100	108.2	0.25
0.15	480	0.022	280	−150	130.5	0.14
0.2	640	0.030	339	−200	139.4	0.06
0.25	800	0.037	388	−250	138.4	−0.01
0.3	960	0.045	430	−300	129.8	−0.05
0.35	1120	0.052	465	−350	115.1	−0.09
0.4	1280	0.060	496	−400	95.7	−0.12
0.45	1440	0.067	522	−450	72.5	−0.15
0.5	1600	0.075	546	−500	46.0	−0.17
0.55	1760	0.082	566	−550	16.9	−0.18
0.6	1920	0.090	585	−600	−14.4	−0.20
0.65	2080	0.097	602	−650	−47.6	−0.21
0.7	2240	0.105	618	−700	−82.4	−0.22
0.75	2400	0.112	631	−750	−118.6	−0.23
0.8	2560	0.120	644	−800	−156.0	−0.23
0.85	2720	0.127	655	−850	−194.5	−0.24
0.9	2880	0.135	666	−900	−233.9	−0.25
0.95	3040	0.142	676	−950	−274.1	−0.25
1	3200	0.150	685	−1000	−315.0	−0.26

Given the reactor power P, we calculate the enthalpy increment per meter $\Delta h = P/(H.q)$.

We calculate $h_{\text{in}} = h_{l,\text{sat}} - z_{\text{sat}} * \Delta h$ and $h_{\text{out}} = h_{l,\text{sat}} + (H - z_{\text{sat}}) * \Delta h$.

Then we calculate the steam mass fraction

$x_{\text{out}} = (h_{\text{out}} - h_{l,\text{sat}}) / (h_{v,\text{sat}} - h_{l,\text{sat}})$,
$\tau_{\text{out}} = (1 - x_{\text{out}}) \tau_l + x_{\text{out}} \tau_v$ and $\rho_{\text{out}} = 1000/\tau_{\text{out}}$.

We assume that $\rho(z)$ is constant for $0 \leq z \leq z_{\text{sat}}$ and then varies linearly from $\rho_{l,\text{sat}}$ for $z = z_{\text{sat}}$ to ρ_{out} for $z = H$.

We calculate its average ρ_{ave} in the pressure tubes and then the coolant effect (in pcm).

Coolant effect $= 2500 * (\rho_{l,\text{sat}} - \rho_{\text{ave}}) / \rho_{l,\text{sat}}$
(In the limit case $\rho_{ave} = 0$ we would get that the coolant effect would be equal to the void effect).

Starting from criticality at $n = 0$, the core reactivity for $n \neq 0$ is computed as the algebraic sum of the (positive) coolant effect and the (negative) Doppler effect. It is computed in column 6 of Table 1. The derivative in pcm/MW is computed in column 7 and plotted in Figure 1.

Finally, our $\alpha(n)$ would be obtained by multiplication of column 7 by 3200.

What it means is that rather than being always negative, $\alpha(n)$ is positive for $n < n_*$ and $\alpha(n) < 0$ when $n > n_*$ with $n_* = 0.238$ that is $P = 762\,\text{MW}$.

Clearly, condition (9) is violated.

In Section 4, we shall prove that, in such a case, the reactor is unstable and we shall study *how* it is unstable.

4 Behavior of the unstable reactor

In this section, we shall assume that there exists n_* such that
$0 < n_* < 1$ and $\alpha(n_*) = 0$
$\alpha(n) > 0$ for $n < n_*$
$\alpha(n) < 0$ when $n > n_*$.
The curve $n \to \rho = \rho(n_0, \rho_0, n)$ that we have defined in (8) is then increasing when $n < n_*$ and strictly decreasing when $n > n_*$ as indicated in Figure 2. Equation (10) may then have 2 solutions: one smaller than n_* and the other one greater. We shall prove that only the latter (which we denote by n_∞) is stable. We shall prove that

- if $\rho_0 > 0$ then $n(t) \to n_\infty$ when $t \to \infty$,
- if $\rho_0 < 0$ and $n_0 > n_*$, then $n(t) \to n_\infty$ when $t \to \infty$,
- if $\rho_0 < 0$ and $n_0 < n_*$, then $n(t) \to 0$ when $t \to \infty$.

Let us first consider the case where $\rho_0 > 0$.

Let us choose $n_1 > n_*$ such that $\rho(n_0, \rho_0, n_1) \geq \rho_0$ (see Fig. 2) and then

$$\rho(n_0, \rho_0, n) \geq \rho_0 \text{ for } n_0 \leq n \leq n_1.$$

From (12) we deduce that

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathrm{Log}\, n \geq \frac{\rho_0}{\tau}$$

and then, as long as $n(t) \leq n_1$

$$\mathrm{Log}\, n - \mathrm{Log}\, n_0 \geq \frac{\rho_0}{\tau}.t$$

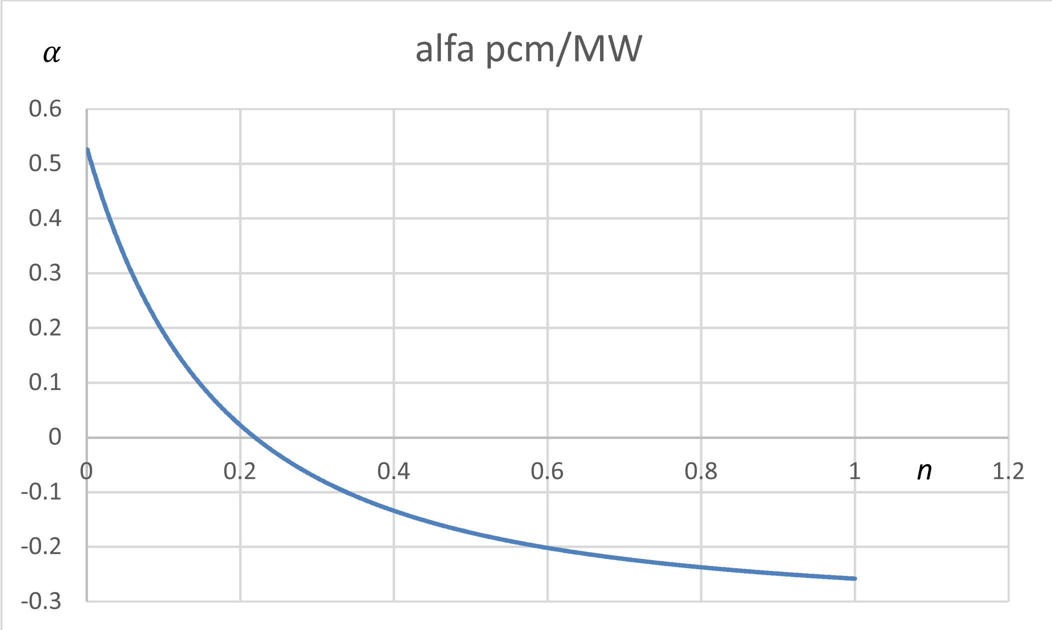

Fig. 1. $n \to \alpha(n)$ for the RBMK before the accident.

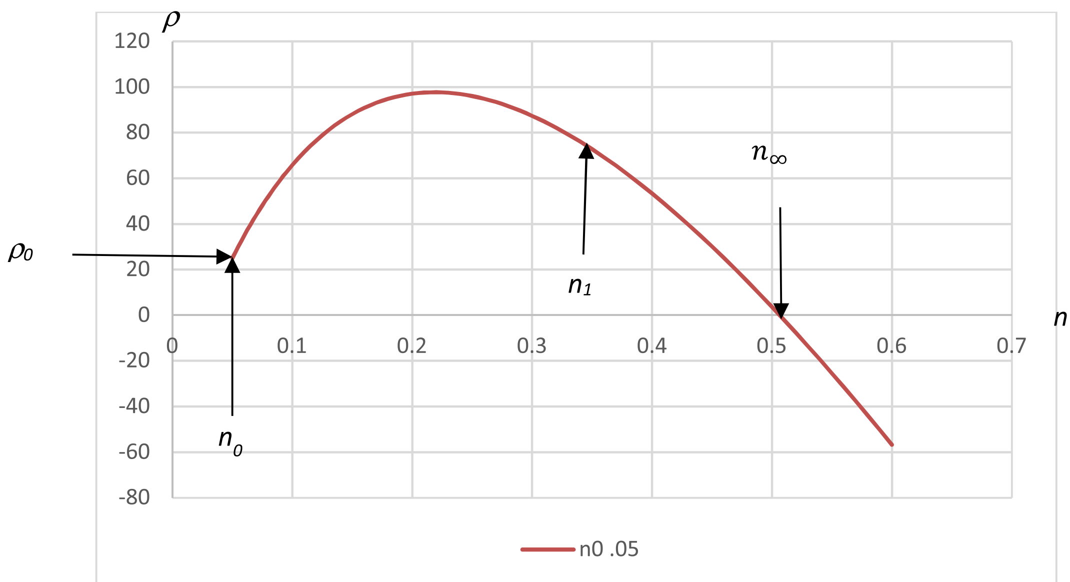

Fig. 2. Example of curve $n \to \rho = \rho(n_0, \rho_0, n)$.

$$n(t) \geq n_0.\exp(\rho_0.t).$$

Then the set of real numbers t such that $n(t) \leq n_1$ is bounded. Then, there exists a time t_1 such that $n(t_1) = n_1$.

Let $\rho_1 = \rho(n_0, \rho_0, n_1)$; we have $\rho(t_1) = \rho_1$.

Let $\alpha_* = min\{\alpha(n){:}n > n_1\}$ with the above assumptions, we have $\alpha_* < 0$.

Since $\alpha(n) < \alpha_* < 0\,\forall\, n > n_1$ we can apply proposition 1 which tells us that $n(t) \to n_\infty$.

The case where $\rho_0 < 0$ and $n_0 < n_*$ is illustrated in Figure 3.

Equation (12) shows that n decreases and hence ρ also decreases. We see that $\mathrm{Log}\, n \to -\infty$ hence $n \to 0$.

To be complete we should also consider the case where $n_0 > n_*$ and $\rho_0 < 0$.

We can just apply proposition 1, which shows that $\rho(t) \to 0$ and $n(t) \to n_\infty$.

We have thus proved the desired result.

Note that we have proved that the starting points $n_0 < n_*$ with $\rho_0 = 0$ are unstable:

- if rather than being equal to zero, ρ_0 is slightly positive we shall have $n(t) \to n_\infty > n_*$.
- If ρ_0 is slightly negative we shall have $n(t) \to 0$.

Note also that with 3 starting points $n_{01} = 0.05$, $n_{02} = 0.12$ and $n_{03} = 0.5$ and the same reactivity

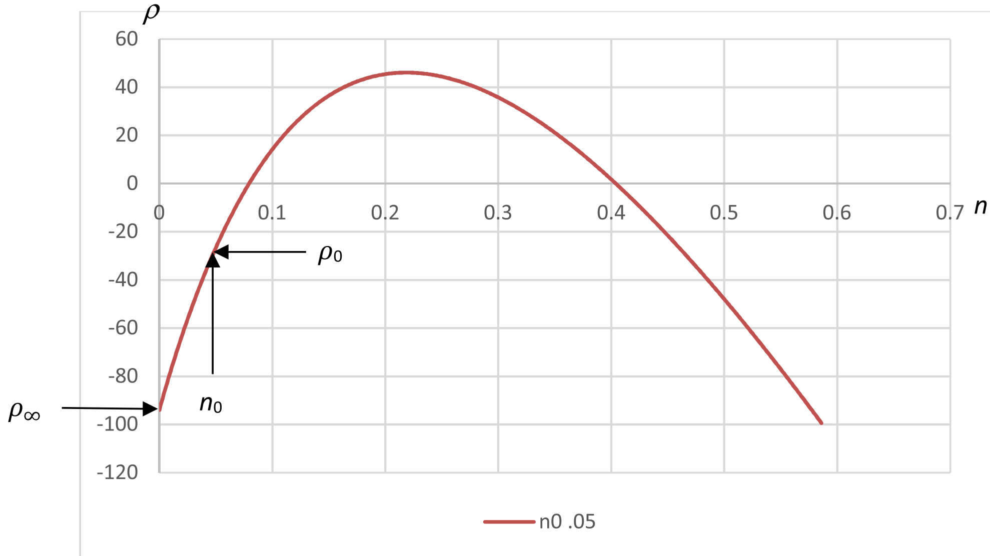

Fig. 3. Curve $n \to \rho = \rho(n_0, \rho_0, n)$ obtained for $n_0 = 0.05$ and $\rho_0 = -26.6\,\text{pcm}$.

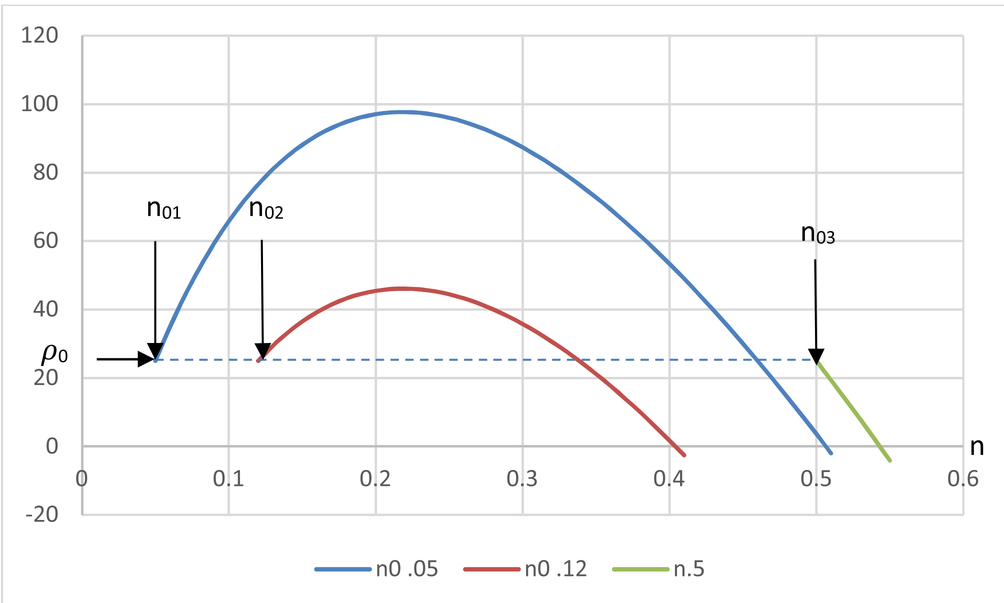

Fig. 4. 3 starting points n_{01}, n_{02}, n_{03} with the same $\rho_0 = 25\,\text{pcm}$.

injection $\rho_0 = 25\,\text{pcm}$, as can be seen in Figure 4 the final power n_∞ may be larger when the initial power n_0 is smaller.

5 Conclusion

With our simple model, we have shown that reactivity feedback is an important phenomenon.

When the reactor is stable, to increase its power, it is sufficient to make an appropriate reactivity injection ρ_0, but, as the operators know it, it is not necessary to make a negative reactivity injection equal to $-\rho_0$ after this positive reactivity injection.

We have also studied one case of a reactor that is unstable at low power, like the RBMK.

We have shown the practical effects of such an instability:

1) with the same reactivity injection, the lower the initial power then the final power may be higher,
2) the line $\rho_0 = 0 \quad 0 < n_0 < n_*$ is a *bifurcation* line.

   If we start from ρ_0 slightly positive we have $n(t) \to n_\infty > n_*$;
   if we start from ρ_0 slightly negative we have $n(t) \to 0$.

This was the case for the RBMK before the Chernobyl accident: the operators were obliged to frequently use the control rods, and, practically, remaining at low power was not comfortable at all for them. This is one explanation for the accident, but of course, it is not the only one [3,4].

Note that our 2×2 model can only give a qualitative behavior of the reactor; to get accurate kinematics, we would have to adjust the effective neutron lifetime τ or use the 8×8 model with delayed neutrons, as described in [1].

As has been shown in [1], the asymptotic behavior is the same for both models.

Our results explain better why Anatoly Dyatlov has written in https://www.neimagazine.com/features/featurehow-it-was-an-operator-s-perspective/:

"*The nuclear safety department of the power plant (...) measured the fast power coefficient at power levels close to nominal full power, ie in the region where it was negative. The results were used by the operators in their everyday work. The latest data prior to the accident gave a value of minus 1.7 × 10^{-4} ß/MW*".

Conflict of interests

The authors declare that they have no competing interests to report.

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This article has no associated data generated and/or analyzed.

Author contribution statement

1. Introduction B. Mercier – 2. Reactor stability B. Mercier – 3. Evaluation of $\alpha(n)$ for the RBMK V. Borysenko – 4. Behavior of the unstable reactor. B. Mercier.

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