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A simple model to show the effect of counter-reactions

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Abstract. We derive a 2×2 system of non-linear ordinary differential equations to show that the reactor is stable when the temperature coefficient is negative.

1 Introduction

In reactivity accidents, the kinetics is fast and the counter-reaction consists mainly of the Doppler effect. In the Nordheim–Fuchs model (see [1]) the fuel is supposed to be adiabatic (no heat exchange with the moderator, e.g. water for pressurized water reactors (PWR)), which is valid for times of the order of 1s only.

In the present paper, we consider slower reactivity injections, so that the counter reactions consist both of the Doppler effect and the moderator effect. Let us denote by ρ the reactivity injected in the core of a nuclear reactor, the linear theory predicts that if $\rho > 0$ (resp. $\rho < 0$) then the number of neutrons $n\left(t\right)$ increases (resp. decreases) exponentially.

More precisely, to somewhat take delayed neutrons into account, let τ denote the neutron's lifetime corrected by the lifetime of precursors, it is shown in Reuss [2] that when ρ is sufficiently small

$$\frac{\mathrm{d}n}{\mathrm{d}t} \sim \frac{\rho}{\tau}n.$$

So we take

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\rho}{\tau}n \tag{1}$$

as our first equation.

Now, when $\rho > 0$ not only the number of neutrons but also the core power and the core temperature increase. In the same way, they decrease when $\rho < 0$.

Due to the Doppler effect and the moderator effect, the reactivity depends on the temperature. We shall say that the temperature coefficient is negative iff there exists $\alpha > 0$ such that

$$\mathrm{d}\rho = -\alpha \mathrm{d}n.$$

This gives our second equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\alpha\frac{\mathrm{d}n}{\mathrm{d}t} = -\frac{\alpha}{\tau}\rho n. \tag{2}$$

Thus the linear equation (1) is replaced by the non-linear system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\rho\\ n\end{pmatrix} = \begin{pmatrix}-\frac{\alpha}{\tau}\rho n\\ \frac{\rho}{\tau}n\end{pmatrix}. \tag{3}$$

Provided that $\alpha > 0$, we shall prove that $n\left(t\right)$ *has a finite limit when* $t \to \infty$ and that $\rho\left(t\right) \to 0$.

In other words, when the temperature coefficient is negative the reactor is stable.

To increase the reactor power it is sufficient to inject some reactivity in the core e.g. by lifting the control bars, but then the reactivity will decrease to zero naturally without having to lower them.

This result is well known, but it seems that our simple model, which we developed for the pedagogical purpose, had not been studied before like we do here.

2 Preliminary remarks

The curves $t \to \{\rho\left(t\right), n\left(t\right)\}$ are straight lines in the plane $\{\rho, n\}$; in fact

$$\frac{\mathrm{d}n}{\mathrm{d}\rho} = \frac{\mathrm{d}n}{\mathrm{d}t}\Big/\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\frac{1}{\tau}\rho n}{-\frac{\alpha}{\tau}\rho n} = -1/\alpha.$$

Dividing the first equation of system (3) by ρ and the second one by n we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\log\rho\\ \log n\end{pmatrix} = \begin{pmatrix}-\frac{\alpha}{\tau}n\\ \frac{\rho}{\tau}\end{pmatrix}. \tag{4}$$

Obviously, if we start from the quarter plane $\{\rho > 0, n > 0\}$, we stay there.

The same for the quarter plane $\{\rho \left\langle 0, n \right\rangle 0\}$.

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In the quarter plane $\{\rho > 0, n > 0\}$, $\rho(t)$ is decreasing and $n(t)$ is increasing.

In the quarter plane $\{\rho \langle 0, n \rangle 0\}$, $\rho(t)$ is increasing and $n(t)$ decreasing.

Let us denote by $\rho_0 = \rho(0)$ and $n_0 = n(0)$. We let $n_\infty = n_0 + \frac{\rho_0}{\alpha}$. We note that the slope of the line joining $\{\rho_0, n_0\}$ to $\{0, n_\infty\}$ is precisely equal to $-1/\alpha$. We also note that

$$\rho(t) = \rho_0 + \alpha(n_0 - n(t)). \tag{5}$$

In the following section, we shall prove that when $\rho_0 > 0$ then $\rho(t) \to 0$ and $n(t) \to n_\infty$.

When $\rho_0 < 0$ the same result also holds provided that $n_\infty > 0$. Otherwise $n(t) \to 0$.

3 Solution of system (3)

We shall proceed by the separation of variables.

We use (5) to eliminate ρ.

We get

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{1}{\tau} n\left[\rho_0 + \alpha(n_0 - n)\right].$$

That is

$$\frac{\mathrm{d}n}{n\left[\rho_0 + \alpha(n_0 - n)\right]} = \frac{\mathrm{d}t}{\tau}$$
$$\frac{\mathrm{d}n}{\rho_0 + \alpha n_0}\left[\frac{\rho_0 + \alpha(n_0 - n) + \alpha n}{n\left[\rho_0 + \alpha(n_0 - n)\right]}\right] = \frac{\mathrm{d}n}{\rho_0 + \alpha n_0}$$
$$\times\left[\frac{1}{n} + \frac{\alpha}{\rho_0 + \alpha(n_0 - n)}\right] = \frac{\mathrm{d}t}{\tau}.$$

So, by integrating both sides we get

$$\frac{1}{\rho_0 + \alpha n_0}\left[\log(n) - \log(\rho_0 + \alpha(n_0 - n))\right] = \frac{t}{\tau} + B.$$

And, by introducing n_∞:

$$-\frac{1}{\alpha n_\infty}\log\frac{\alpha(n_\infty - n)}{n} = \frac{t}{\tau} + B.$$

To get the value of B, we apply this relation at $t = 0$:

$$B = -\frac{1}{\alpha n_\infty}\log\frac{\alpha(n_\infty - n_0)}{n_0}.$$

Finally, we get

$$\frac{t}{\tau} = \frac{1}{\alpha n_\infty}\log\frac{\alpha(n_\infty - n_0)}{n_0}\frac{n}{\alpha(n_\infty - n)}$$
$$\frac{(n_\infty - n)}{n}\frac{n_0}{(n_\infty - n_0)} = \exp\left(-\alpha n_\infty \frac{t}{\tau}\right)$$
$$n(n_\infty - n_0)\exp\left(-\alpha n_\infty \frac{t}{\tau}\right) = n_0(n_\infty - n)$$
$$n\left[n_0 + (n_\infty - n_0)\exp\left(-\alpha n_\infty \frac{t}{\tau}\right)\right] = n_0 n_\infty$$
$$n(t) = \frac{n_0 n_\infty}{n_0 + (n_\infty - n_0)\exp\left(-\alpha n_\infty \frac{t}{\tau}\right)}. \tag{6}$$

We see that if $n_\infty > 0$ then $n(t) \to n_\infty$ so that $\rho(t) \to 0$.

In this case, the reactor power decreases to a non-zero power.

If $n_\infty = 0$ the formula giving $n(t)$ is undetermined, but by continuity, we can assume that $n(t) \to 0$ and $\rho(t) \to 0$. The reactor shuts down smoothly.

If $n_\infty < 0$ then $n(t) \to 0$ but $\rho(t) \to \rho_0 + \alpha n_0 = \alpha n_\infty < 0$.

The reactor shuts down rapidly.

4 Numerical values (PWR case)

In Section 4, we define $n = n(t)$ not as the number of neutrons but as the fraction P/PN where P is the reactor power of the core, and PN is the nominal power of the core.

We note that if $n_0 = 0$ and $\rho_0 > 0$, then $n_\infty = \rho_0/\alpha$. Therefore to get $n_\infty = 1$, we need to take $\rho_0 = \alpha$. Therefore α is called the *power defect*, i.e. the reactivity to be injected in the core to shift the core power from 0 to PN.

The power defect takes into account the Doppler effect (when n increases from 0 to 1 the fuel temperature increase is about equal to 430 °C) and the moderator effect (when n increases from 0 to 1 the moderator temperature increase is about equal to 10 °C) but also a flow redistribution effect.

The fuel temperature coefficient is about −3 pcm/°C. The moderator temperature coefficient is about −5 pcm/°C at BOC (beginning of cycle) (−60 pcm/°C at EOC (end of cycle)).

Typical values for the power defect are 1300 pcm (i.e. $\alpha = 0.013$) at BOC and 1900 pcm ($\alpha = 0.019$) at EOC (see e.g. [3]).

If we start from $n_0 = 0.1$ and $\rho_0 = 100$ pcm, at BOC, we reach $n_\infty = 0.177$. At EOC we rather reach $n_\infty = 0.153$.

If we start from $n_0 = 0.1$ and $\rho_0 = -100$ pcm we get $n_\infty = 0.023$ at BOC (0.047 at EOC).

A typical value for τ is $\tau = 0.08$ s at BOC.

Applying formula (6) with $t = 100$ s we get $n(t) = 0.049$ and (applying (5)) $\rho(t) = -34$ pcm which means that the decrease is rather slow.

5 A more accurate model

To get a more accurate model we should start from the six groups model (see Reuss [2]) i.e., the following 7×7 differential system:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\rho - \beta}{\ell_0}n + \sum_i \lambda_i c_i \tag{7}$$

$$\frac{\mathrm{d}c_i}{\mathrm{d}t} = \frac{\beta_i}{\ell_0}n - \lambda_i c_i \; 1 \le i \le 6 \tag{8}$$

where $n = n(t)$ is the number of neutrons and $c_i = c_i(t)$ the number of precursors, ℓ_0 the neutron lifetime, λ_i the associated decreasing constant °, and $\beta = \sum_i \beta_i$ the delayed neutrons fraction.

For uranium-based fuel, numerical values of these parameters can be found in reference [1].

To take counter-reactions into account we decide that ρ is the 8th variable and we complement this 7×7 system with an 8th equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\alpha \frac{\mathrm{d}n}{\mathrm{d}t} \cdot \tag{9}$$

We also specify some initial conditions:

$$n\,(0) = n_0; \quad c_i\,(0) = c_i^0; \quad \rho\,(0) = \rho_0.$$

We can expect, like above, that $\rho\,(t) \to 0$ when $t \to \infty$ and also (taking $\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\mathrm{d}c_i}{\mathrm{d}t} = 0$ in (7) and (8)) that

$$n\,(t) \to n_\infty = n_0 + \frac{\rho_0}{\alpha}$$
$$c_i\,(t) \to \frac{\beta_i}{\ell_0} n_\infty.$$

Note that this 8×8 nonlinear differential system is implemented in nuclear reactor simulators, and the numerical results show that the reactor is stable provided $\alpha > 0$.

So, maybe the kinematics is not the same as in our simple model, *but, if* n_0 *and* ρ_0 *are the same, the limit* n_∞ *is the same.*

Thus, we think that our simple model will be useful for pedagogical purposes.

6 Conclusion

We have derived a simple model which represents qualitatively the reactor behavior and may make the students better understand the effect of counter-reactions.

Conflict of interests

Paul Reuss and Bertrand Mercier declare that they have no competing interests to report.

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Author contribution statement

Paul Reuss found the analytical solution to system (3), which is the core of the paper. The remaining parts originated from Bertrand Mercier.

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